Endogenous Schumpeterian Growth, The Productivity Slowdown, and Entrepreneurial Dynamics

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ABSTRACT. This paper used concepts from chaotical dynamic systems, information theory, and cybernetics to develop a model that depicts a Schumpeterian process through which entrepreneurs search for new combinations of both products and production processes that result in increased productivity and growth. The model provides the basis for an alternative conceptual framework for conducting research into the causes of the productivity slowdown that began in the United States in the early 1970s and continued until recently.

Introduction

Economic theory has focused increasingly in recent years on developing a deeper understanding of the mechanism of economic growth, in response to an awareness that conventional neoclassical growth models shed little light on modern growth problems. High on the list of such problems is the productivity slowdown that has occurred in advanced economies from the early 1970s until recently. Many studies, following Romer (1986, 1987, 1994), have focused on the development of endogenous growth models that explain growth by employing the broad assumption that information spillovers constitute a productivity-enhancing externality (Shell, 1966; Arrow, 1962; Judd, 1985; Grossman and Helpman, 1989). This assumption, however, simply shifts the explanation of the process of technological change from an undefined "exogenous" factor to another undefined element, "information." These models link technological advancement to information without defining information or showing how information arises or explaining its specific link to productivity and growth.

This paper presents an endogenous model of technological change and productivity growth that incorporates linkages between entrepreneurial firms, information, and technological change. This model presents an alternative framework for research into the causes of the decline of productivity growth rates that began in the early 1970s (Barro and Sala i Martin, 1992). It uses concepts from the modern disciplines of chaotical dynamic systems, formal information theory, and cybernetic theory, which have been employed increasingly in economic analysis (Baumol and Wolf, 1992; Boldrin & Deneckere, 1990; Jenner, 1966, 1991, 1994), to represent an aggregate production function as a dynamic, stochastic system.

The concept of wealth

It has been long recognized in the literature that defining growth simply as an increase in GDP contains many ambiguities and tautological problems (Hogendorn, 1987, pp. 30–34). Conventional economic analysis has long held that wealth is essentially a quantity of goods that satisfy human "needs" or "wants" (Marshall, 1920, pp. 41 and 47). The exact meaning of "wants" or "needs," or a deeper analysis describing what they are and ways in which goods meet them, has been largely ignored in the literature.

In this analysis, I shall replace the terms "wants" and "needs" with "individual or consumer objectives." In this approach, the "utility" of a good represents the extent to which consumers use

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it to achieve their personal objectives, which ultimately reflect their values and individual goals. I shall assume that every person has a unique set of personal goals that reflect individual differences in lifestyles, tastes, incomes, and aspirations.

From this viewpoint, wealth, $\hat{W}$, may be roughly approximated as a positive function of the number, or variety, of distinct product choices that are made, on average, by consumers in a society during a specific time interval. Moreover, individual utility, considered as the ability of any given product choice to best suit the individual objectives of each consumer, in general, is increased when the aggregate variety of products, including the number of distinct models and variations of each product, as well as the number of goals or objectives for which each product may be used, is expanded. Broadly speaking, the greater the number of choices available, the greater the likelihood that each consumer will be able to find the set of choices that is specifically tailored to his or her individual goals or objectives. In other terms, increases in wealth result in an increase in control by individuals. As Arthur Lewis once wrote, "What distinguishes men from pigs is that men have greater control over their environment, not that they are more happy" (Lewis, 1955, p. 421).

Innovations that create wealth

Following the pioneering work of Abernathy and Utterback (Abernathy and Utterback, 1987; Abernathy and Clark, 1985; Utterback, 1994), many students of innovation and technological change identify two distinct, but interrelated, phases connected with the innovative process.

The first phase begins when entrepreneurs perceive that technological change has opened an opportunity to develop a new product. Technological change continually generates new products, components, and production techniques from which, as Schumpeter noted (1934), new and innovative combinations can be assembled. The question is, of course, what form should the new product combination take, and for what uses (or objectives)?

This phase is characterized by intensive search, typically by many new enterprises that enter the nascent "industry," for the configuration of product features that will be most preferred by customers. At this stage, which represents the "birth" of an industry, considerable innovation is directed toward combining components, features, and designs in a way that will appeal to the largest number of consumers. Although the essence of this activity is the Schumpeterian process of assembling "new combinations" of existing products (Schumpeter, 1934), it also requires considerable modifications of the components that comprise each new combination.

The result is the development of a "dominant design" (Abernathy and Utterback, 1978; Utterback, 1994), which is clearly preferred over all other previous and current product versions. For example, after several years of industry experimentation with different typewriter designs, the Remington "Number 5" model emerged in 1899 as the dominant design; its combination of central features, including a tab, visible type, quiet operation, and ease of correction, were incorporated in all subsequent typewriter models (Utterback, 1994, pp. 24–26). It is evident that the emergence of the dominant design results in the maximum amount of new wealth, as we have defined that concept, that can be obtained from new combinations of features and components. Once the dominant design appears, the center of gravity of competition shifts to a search for innovations that increase the efficiency of the production process. Typically, only a few entrepreneurial firms, those in the strongest position to produce and market the new dominant design, survive to enter the new stage of competition, and most of the firms that were involved in the search for the dominant design leave the industry. Each of these growth stages will be considered in turn.

Search for a dominant design

In the flow of new products and product variations that appear in the economy, triggered in most cases by a cascade of entrepreneurial small businesses, at some point a potential new product emerges. This new product, as Schumpeter noted, is largely the result of new combinations of existing products, as well as new combinations of existing production paradigms.

In describing this and subsequent stages, I will use the term "basic transformation" to represent any sequence of tasks that changes the form or structure of a set of inputs to produce outputs. In addition, "technological driver" will designate a group of basic transformations that are combined to form an interactive pattern of change that results in the production of wealth. Although basic transformations may not always be embodied in physical form or equipment, a technological driver represents a pattern of changes that has a material effect on the choices available to consumers. For example, as Peter Drucker has noted, the modern computer, which is a technological driver, was made possible – almost inevitable – by several basic transformations: the invention of binary arithmetic by Gottfried Leibniz, the development of the "computational machine" by Charles Babbage, the discovery that logical statements could be represented in arithmetical form by Whitehead and Russell, and the invention of the audion tube by Lee DeForest (Drucker, 1993, pp. 24–25).

As entrepreneurs sense that a new technological driver might be formed from a set of available basic transformations, they begin a search process. As we have seen, this is essentially a search for a new dominant design, which we will represent by the function:

$$f(x_k) = \alpha\lambda x_k(1 - x_k) \qquad (1.0)$$

where x_k represents a specific and novel combination of components that define a potential new product that is capable of generating new aggregate wealth, $\hat{W}$,. In order to simplify the presentation, all values of x are presented in proportional terms to yield a maximum value for x of 1. This simplification is strictly for mathematical convenience and in no way limits the result of the analysis (see Bergé, Pomeau and Vidal, 1984, p. 196).

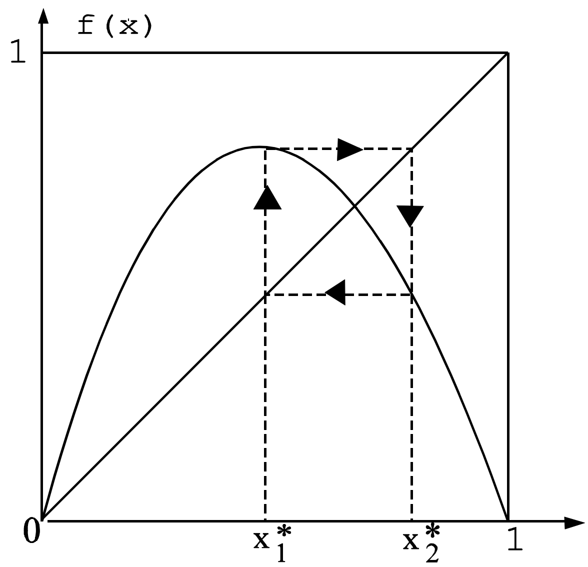
In addition, this representation assumes that during a given time interval the total number of products and potential product combinations other than those involved in developing the dominant design in this industry, or for this new product variant, is constant. Each x_k represents a specific combination of these existing features and components that defines a unique product form.

In Equation 1.0, α represents the number of firms competing or presenting each component combination or product variant during each search period, and λ represents the total number of new combinations of products, features, designs, and components (including materials) that all firms bring to the market during each iteration. Specifically, λ is determined by two factors:

1. The variety of sources of capital, such as venture capital, bank loans, private placements of all types, and the willingness of existing firms to enter into joint or collaborative ventures. All such opportunities may be considered as channels or pathways that are available to entrepreneurs to develop, produce, and market a new product. These "channels," of course, are substantially affected by the stability of regulations, public policies, and governmental administrative rules.

2. The capacity (also defined as "entropy") of available information or communication channels. The number of channels includes all means by which information concerning technological options is disseminated; the entropy of such channels is also affected by the regulatory and public policy environment.

Equation 1.0 presents the essential dynamics of the search process represented in Figure 1. The parabolic curve, or "subharmonic cascade," shown in Figure 1, which has been adapted from Bergé, Pomeau and Vidal (1984, p. 195), represents the transformation, or "mapping," of x_k into x_{k+1},



(Adapted from Bergé, Pomeau and Vidal, 1984, p. 197)

Figure 1. The subharmonic cascade.

given by Equation 1.0. Selected equilibrial values of x in this equation are shown in Figure 1. This graph depicts what is termed the "first return map" of the function shown in Equation 1.0, plotted against a 45° line from the origin. The height of the parabola in Figure 1 increases as $(\alpha * \lambda)$ increases, i.e., as the number of competing products and the number of design variations that these products contain increases.

It can be seen that in Figure 1 the function f will transform any arbitrary value of x into x_1^* and x_2^*, both of which represent temporary equilibrium values or forms of the product (except, of course, for $x = 0, 1$, where no products are introduced or developed). Thus the function will "map" any x with a value less than x_1^* into a new, larger value, i.e., the distance above the 45° diagonal, the value of which is then shifted horizontally to the diagonal. Such "iterations" continue until x is equal to x_1^*, which is an equilibrial value. Similarly, f causes values of $x > x_2^*$ to contract, ultimately to x_1^* or x_2^*. Thus, if a product version appears for which the value of the initial equilibrial design is x_1^*, the function then maps this product into x_2^*. The two values of x, therefore, constitute an alternating sequence, which is called in dynamic theory a two valued "attractor" of the mapping function.[1]

Each equilibrium value represents a specific design of the new product that is put into the market and replaces the product variation that had achieved equilibrium in the previous period. More specifically, when an "equilibrial design" appears, it results in greater sales than did the previous period's equilibrium product.

Figure 1, therefore, represents a mapping function with specific values for parameters α and λ for which two equilibrial designs are developed by entrepreneurs and introduced into the marketplace. This depicts the situation where firms seeking to increase their competitive advantage consider only two product combinations. New firms entering the market will only introduce copies of one or the other of the two variants.

For low values of parameters α and λ, the search process explores only one or a few values of x. However, as $(\alpha * \lambda)$ increases, the number of firms entering the market and the number of variants both increase, and successive mappings or transformations are essentially "chaotical," or unpredictable and random (Collet and Eckmann, 1980). The result is that all possible values of x – all of the potential combinations of components – shown on the horizontal axis are successively tried in the marketplace by entrepreneurial firms.

Therefore, as the magnitude of the parameters in Equation 2.0 increases, more product combinations are tried, and a greater number of values of x are explored, until one becomes dominant and results in the greatest net increase in wealth for consumers. At this point, firms that are not well situated to exploit the new dominant design exit the market, parameters α and λ fall, the parabolic function, f, shifts downward, and the search function settles on a single value of x – a single form of the product – which then becomes the dominant design. Even after the dominant design emerges, some firms may continue to experiment with new combinations, but none will succeed in replacing the dominant design.

Equation 1.1 shows the process that determines the development of the dominant design:

$$\dot{w} = \sum(v'y' - vy)H(Y_k)_t \tag{1.1}$$

where $\dot{w}$ is the rate of change of wealth. As each new product version, Y_k, appears in the market, the change in "wealth" is represented by $\sum(v'y' - vy)$, where v' and v describe the wealth achieved by any new product and the wealth or value that characterized the product that the new product replaced, respectively. $H(Y_k)_t$ shows the entropy of the search process, given in Equation 1.0 and Figure 1.

The emergence of the dominant design is linked to the search process (Equation 1.0) by

$$\lambda = a[\sum(u'v' - uv)H(x)] + 1$$

$$\alpha = b[\sum(u'v' - uv)H(x)] + 1$$

When $\dot{w} = 0$, new product variants generate no new wealth, and the product that had become "equilibrial" in the previous period becomes the dominant design.

At this point, the search process to develop a dominant design stops, firms leave the market, parameters α and λ decrease to critical values, and the pattern of innovation shifts to the development of process improvements.

Search for production process innovations

A production system that transforms inputs into outputs by performing a series of production tasks, which are executed by labor and capital factors of production, may be represented as a stochastic, "theoretical machine," as employed in cybernetic analysis (Jenner, 1994a, 1994b). Production tasks are any activities performed by workers in conjunction with capital goods, such as machines and tools, that result in a value-added change in any set of inputs or workpieces in a production system.

In general, production tasks are performed sequentially upon each set of inputs that moves through the production system; they may be represented as a permutation, or, as designated in cybernetics, a protocol, $P = (t_2, t_2, \ldots, t_k)$, where each task, t_k, is replaced by the one to its right, and the last task is followed by the first.

Some tasks, such as handling, moving, and positioning workpieces, are primarily performed by labor; other tasks, such as drilling, milling, planing, and welding, are accomplished principally by tools and machines assisted by labor. In general, workers perform two broad types of essential tasks in the production process. One type, which might be called "primary tasks," directly transforms inputs or workpieces. The second type of tasks consists of corrective tasks, such as adjusting tolerances of cutting tools, recalibrating settings, repositioning workpieces, to compensate for variations in inputs. Many of these corrective tasks are required because of errors in the execution of production tasks at earlier or current production stages. When unwanted variation in output occurs, compensating variation in production tasks is required to eliminate it. This result is demonstrated rigorously in the cybernetic "law of requisite variety" (Ashby, 1964; Jenner, 1966).

Finally, variations in primary production tasks also contribute to unwanted variations in output. These tasks may be mistakes in the production process, poorly executed or performed in error, or they may no longer be suited to the configuration or structure of these inputs because users of this product, in the case of intermediate goods, have changed their production processes.

In general, therefore, the model of the production system must represent three types of production tasks: primary tasks, tasks intended to correct or adjust for variations in inputs, and tasks that generate undesired variances on the workpieces.

Each protocol of production tasks that transform a set of inputs to produce a specific output defines a production stage. Since the order in which tasks are performed depends upon the random appearance of deviations in tasks and workpieces in each stage, the sequencing of production tasks may be effectively represented as a stochastic function: (ρ_{ij}), where ρ_{ij} represents the frequency with which the ith production task (including all three types of production tasks) is followed by the jth task.

The total entropy, $H(\rho)$, or variety, in a production system with i stages can be represented by:

$$H(\rho_i) = \sum_i \left[\left(\sum_j \rho_{ij} \right) \tau_i \right) \tag{2.0}$$

where τ is the long-term frequency with which each of the i stages will appear, the "equilibrial proportions" that are solutions to the stochastic matrix (Ashby, 1964, pp. 175–176).

As the number of tasks needed to produce one unit of a product increases and the transition probabilities for these tasks become more nearly equal, i.e., their "scatter" increases, entropy rises. If the average time needed to accomplish a production task is constant, an increase in entropy will reduce output for any given time period. More generally, we can say that the efficiency of labor, ε_L, is given by Equation 3.0:

$$\varepsilon_L = \hat{W}_A / H(\rho)_L \cdot t_\rho \tag{3.0}$$

where $\hat{W}_A$ represents the wealth embodied in each unit of product, A, $H(\rho)_L$ the entropy of production tasks performed by labor, and t_ρ the average time it takes labor to perform these tasks.

The search for the most efficient production technique begins as soon as the dominant design emerges. This exploration may be represented as a subharmonic, chaotical cascade, as depicted in Figure 1. Here, however, the values of x are combinations of production goods for which $H(\rho)$ is low.

Shift to productivity innovations

Because production technologies shift constantly as new product features are introduced, firms engaged in the search for a dominant design emphasize the use of skilled labor, which provides maximum flexibility. Once the dominant design has been developed, however, innovative improvements in production systems, illustrated by the search function depicted in Figure 1, result in reduced production system entropy, and consequently in lower prices, and, ultimately, increased sales and the diffusion of the new dominant design throughout the economy. Innovations that improve the efficiency of production processes are the most significant causes of (labor) productivity improvement. For example, fabricating the bulbs for Edison's dominant design for the (carbon filament) incandescent light bulb initially was a highly labor-intensive process, requiring more than 200 labor-intensive steps (Utterback, 1994, p. 71; Bright, 1949). However,

> . . . a string of process innovations would dramatically reduce production times, reduce the skill and experience needed by labor, and introduce a number of lamp-making machines that together would cause the price of the standard electric lamp to drop precipitously (Utterback, ibid., p. 71).

One such innovation, the Sprengel mercury pump, reduced the time needed to remove air from each bulb from five hours to thirty minutes.

Searching for efficient techniques to produce the dominant design centers on developing a "productivity driver," through a search process similar to that shown in Figure 1. The productivity driver represents new possibilities, new arrangements of existing products and processes, that have the potential to increase the efficiency of the production process used to make the new dominant design. Production efficiency increases when the new driver (embodied in a single machine or a set of complementary machines, devices, and implements) has the potential to (1) replace tasks in the current production process, (2) eliminate unwanted tasks in the current process, (3) reduce the variability of intermediate and final outputs, and (4) decrease the average time needed to accomplish a set of production tasks, typically a function of capital used in the production process.

Since a production system consists of production stages that are coupled through input/output flows, introducing a new piece of capital equipment in any stage of a production system generates changes that spread throughout all stages of the system (Jenner, 1994b; Senge, 1990). In the short term, at least, all these changes result in increased production entropy and a corresponding decline in efficiency.

Let $\hat{z}$ represent all inputs to the production process used to produce the dominant design, x. The search process can be represented as a subharmonic cascade:

$$\hat{z}_{k+1} = \delta\mu\hat{z}_k(1 - \hat{z}_k) \tag{3.1}$$

In Equation 3.1, $\hat{z}$ represents the improvement in ε_L, the gain in efficiency provided by each new technique.

δ represents the number of combinations of components and machines that are considered during each period of the search phase. δ is influenced by perceptions of risk, availability of capital, flow of information about available alternatives and willingness of existing or potential firms to experiment.

Finally, μ represents the change in entropy of the production process that employs the new productivity driver. These changes, as we have noted, occur because the introduction of the new driver in any stage of the production process requires corresponding changes in all components of the system connected to that stage. These changes constitute unwanted variations in inputs and outputs and are disturbances that require corrective tasks, and even equipment. The results are ripples of change that spread through the system chaotically and reduce production efficiency.

As is true of the search process for a dominant design, as the parameters δ and μ in Equation 3.1 increase, the number of potential solutions increases to an intensity characterized by chaos; many products and protocols are developed, but none confer significant increases in production efficiency. On the other hand, as both δ and μ converge toward unity, the search process converges on a single dominant version for a productivity driver, and efficiency increases.

An important exception to this process occurs when new technology results in sufficient reduction in production entropy to outweigh any change in entropy in other stages of the production function in which it is introduced or any increases in supplier entropy and costs.

Realignment of factors

Each new productivity driver is joined with a configuration of complementary machines, components, and implements; together they define a new production protocol, ϕ, which is selected through a subharmonic search process as shown in Figure 1.

Let $\varepsilon(\phi_k)_j = \hat{H}(\phi_k)_j - \hat{H}_D(\chi_i)_j$ represent the gain in efficiency within the *j*th production system that is caused by introducing the new production protocol ϕ in the *k*th stage of that system. Here, $\hat{H}(\phi_k)_j$ is the entropy of the *j*th production system after the introduction of the new production protocol ϕ; $\hat{H}_D(\chi_i)_j$ is the efficiency of the jth system before the new protocol ϕ_k is substituted for χ_i. In many cases, as we have seen, the initial introduction of ϕ causes a ripple of disturbances throughout the production system, and $\hat{H}(\phi_k)_j > \hat{H}_D(\chi_i)_j$.

For *j* firms that introduce protocol ϕ, we have:

$$\sum_j \varepsilon(\phi_i)_j = \sum_j \hat{H}(\phi_i)_j - \prod \hat{H}_D(\chi_i)_j \qquad (3.2)$$

where the last term describes feedback-induced chaotical dynamic changes caused by the introduction of the new protocol. Because of the resulting increased entropy, introducing the protocols in Equation 3.2 during this period elicits responses, as the firms attempt to control the nonlinear fluctuations that result. Some of these responses are new technological and productivity drivers derived from sources external to the industry, ψ_l. Some responses consist of changes in the features (or component combinations) of the dominant design, $\Delta\lambda$, and still others are modifications on existing production protocols throughout the firm affected by the chaotic change, $\Delta\phi$. Throughout all *j* industries, entropy is reduced only when:

$$\sum_j \hat{H}(R_1) > \sum_j \hat{H}_D(\chi_i)_j$$

where $\hat{H}(R)$ is the entropy of responses and $\hat{H}_D(\chi_i)_j$ the entropy of all disturbances.

The group of *j* firms constitutes a single, interrelated entity, in which the high degree of interactivity is caused by a rich network of feedback loops that spawn chaotic activity.

In many cases, production-process innovations initially tend to increase entropy and reduce production efficiency because they require new types of inputs, with new specifications, often with greater volume and speed of throughputs than before. These factors put stress on all production stages and protocols within the firm introducing the new protocol and within other firms connected to it, and force these firms to align their production protocols to accommodate the innovation. Both δ and μ tend to increase, and, as Equation 3.1 shows, no efficiency driver becomes equilibrial.

The process of reducing entropy, and thereby increasing production efficiency, requires simultaneous and carefully correlated changes in production stages within each firm and between all firms that form a single coherent grouping based on input/output flows. This, in turn, requires effective communications channels with a capacity that is at least as broad as the variety, or entropy, of the disturbances. Moreover, the fluctuations in a chaotical system, as Shaw (1981) has shown, generate new information, ideas, and combinations throughout the search process. Therefore, chaotical patterns occur when simultaneous changes are coordinated throughout the system through nonlinear fluctuations, forming what Prigogine and his colleagues call a “chaotic dissipative structure.” (Prigogine and Stengers, 1984; Jenner, 1989, 1991, 1994a, 1994b). This requires:

1. Information about new technologies must flow efficiently both between and within all firms so all production protocols can develop and make simultaneous changes that complement each other. When this occurs, all production protocols forming the new highly coupled dissipative structures are aligned in the sense that any change in productive tasks increases systemic entropy.

2. A wide variety of pathways must exist through which new enterprises may emerge. Clearly, lack of venture capital may impede experimentation with more efficient production processes.

3. A nation’s regulatory environment provides a basic framework within which these communications channels function by specifying which options are permissible, by directly prohibiting certain types of cooperative efforts, and by establishing rules under which all firms function.

As we have seen, when a new process technology is introduced into a production system, it creates the need to adjust, or realign, other production tasks in other stages of the system and in the wide network of other firms with which the initial system is linked. The result is rising entropy, which increases μ and λ in Equation 3.1, which increases the intensity of the search process and ends in chaos.

Summary and conclusion

This paper has developed a model that depicts an essentially Schumpeterian chaotical search process through which new products and production technologies are developed. In this model, when entrepreneurial experimentation causes a cascade of potential new product combinations to appear, trial-and-error experimentation generates information that may determine the exact configuration of features and components in new products, production methods, and capital goods. This activity, driven most frequently by new small entrepreneurial firms, and which includes a substantial element of "positive feedback," exhibits many characteristics of chaotical systems. Through this process, alternative combinations of products are developed, information is generated, and a "dominant design" emerges. When a dominant design evolves firms create productivity-enhancing capital goods and investments.

Efficiency and productivity improvements result when discontinuities are removed from a production process, when the process resembles a "flow." This occurs when all components within an organization, an industry, and a national economy, are "aligned" to a common focus and aggregate entropy is minimized. In general, productivity increases when innovations are directed toward reducing steps or tasks in production processes. These innovations tend to mechanize production by replacing labor tasks, shortening the time needed to execute a task, or reducing the number of tasks.

Finally, the emergence of a new product or new production method in one industry spreads change chaotically throughout an economy, through positive and negative feedback loops that create potential new consumer products and production methods. In turn, new production methods may induce the formation of new products in entirely different industries.

The model presented in this paper provides the basis for an alternative conceptual framework for conducting research into the productivity slowdown in the United States. It suggests, for example, that the new environmental standards that were applied to businesses in many industries in the 1970s, may have played a role in this phenomenon. For example, many firms may have diverted more resources and time searching for products and processes that met the emerging new environmental standards, but did not generate, in the conventional sense, new "wealth" or greater productive efficiency.

Other factors may have played a role, such as the sharply increased levels of global competition, particularly from Japan, that appeared in the late 1960s and early 1970s, and the twin "oil shocks" that occurred during the 1970s. Increasing global competition through imports of largely standardized commodity products increased the risk of, and reduced the expected gain from, process innovations, which discouraged innovation and investment in these goods. New entrepreneurs, facing a high degree of uncertainty concerning the future direction that environmental standards, and the new production and product technologies and innovations began to concentrate their efforts on services and design or knowledge-intensive fields, such as computer software and associated products, where conventional measurements of growth present substantial conceptual difficulties (David, 1990; Zuboff, 1988).

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Note

1 It can be shown that x_1^* and x_2^* represent fixed points of the function $f^2(x)$ (Bergé, Pomeau and Vidal, 1984, pp. 196–197).

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